On the Ending Rule in Sequential Internet Auctions

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Abstract

This paper shows why having a fixed end time instead of a flexible end time is desirable in sequential Internet auctions. There are two multi-round second-price private-value auctions run in a sequence. When all the buyers bid in the first auction, the ordering of their valuations is revealed. Different positions in the ordering give different motives for further bidding in the first auction. The strong buyer prefers to shade the value, so that he does not have to pay too high a price. The weak buyer prefers to bid the valuation, because he has no chance for a positive transaction in the second auction. Altogether, they cannot reach an equilibrium in which they learn the ordering of their valuations before the last round of the first auction. In equilibrium, they bid in the last round of the first auction. This gives rational for sniping, a common practice of placing a bid in the closing seconds of the auction, and proves that the flexible end time is not the best solution.

Key words: Ending Rule, Sequential Auctions, eBay, Sniping
JEL classification: D44

1 Introduction

Different ending rules are used in Internet auctions. On eBay, the seller specifies the fixed end of the auction. All bids must be placed within the auction duration. It is possible to snipe, that is to place a bid in the closing seconds of the auction, so that the other buyers have no time to react. On Amazon, the seller determines the length of the auction, but the end time is flexible. Whenever a bid is submitted in the last 10 minutes of the specified auction duration, the auction is automatically extended for an additional 10 minutes from the time of the latest bid. It is always possible to react to the late bid. On Yahoo, the seller can opt for the flexible end time or choose to have the fixed end time.

Internet auctions with different ending rules have been studied in the literature, but none of the existing theories is fully consistent with the empirical findings. Bajari and Hortacsu (2003) as well as Roth and Ockenfels (forthcoming) neglect the presence of multiple auctions selling similar goods (see Anwar et al. (forthcoming) for empirical evidence of bidding across the competing auctions). On the other side, Peters and Severinov (forthcoming) model competing auctions. They show that incremental bidding is an equilibrium strategy. Contrary to their predictions, the experienced on-line bidders learn to avoid incremental bidding (see As Roth and Ockenfels, forthcoming).

This paper models competing private value Internet auctions with and without a fixed end time. The model of the Internet auctions with the fixed end times is similar to the sequential sealed-bid second-price auctions, as modeled by Milgrom and Weber (1982) and Weber (1983). Two auctions selling the same good are run in a sequence. Buyers have unit demands. Each auction has a finite number of rounds. In each round, a buyer can submit a bid. After the final round is reached, the auction ends. By early bidding in the first auction, buyers reveal information on their valuations. The weak bidder learns that he has no chance to win the good in the second auction. Therefore, he weakly prefers to increase his bid in the first auction. There is an efficient equilibrium in which he does not increase his bid. This is possible, as for some of the opponents' strategy profiles, he is indifferent between increasing his bid and not bidding at all. Yet, his behavior is not fully "rational", as he does not do what he weakly prefers to do. In a more "plausible" equilibrium, he always increases his bid, when he learns that he has no chance to win the good in the second auction. When he does so, the other buyers never want to let him know that he will actually not win any good in the second auction. Since their bids might reveal this information, they prefer not to bid before the last stage of the first auction, so that the weak buyer has no time to react. In a more "plausible" equilibrium, buyers always snipe. Hence, they behave as is commonly observed in Internet auctions with fixed end times (see Bajari and Hortacsu, 2003 and 2004, Roth and Ockenfels, 2002 and forthcoming, Wilcox, 2000).

In the model of the Internet auctions with flexible end times, all the rules are the same as in the case of the fixed end time, but the final round of an auction is modeled differently. Whenever a bid is submitted in the last round, the auction is automatically extended for another round. As under the fixed end time, after seeing the early bids, the weak buyer weakly prefers to increase his bid. There is an efficient equilibrium in which he does not increase his bid, but it is not really a plausible equilibrium. Moreover, there is no "plausible" efficient equilibrium. This is because under the flexible end time buyers are unable to wait to bid for the last round of the auction. All in all, the fixed end time is necessary for the "plausible" market equilibrium. Interestingly, on Yahoo, sellers seem to know the answer, as they usually choose the fixed end time, when selling private value goods (see Schindler, 2003). This again indicates that the model performs well empirically.

The paper is structured as follows. The next section studies sequential Internet auctions with flexible end times. Subsection 2.1 introduces the model of the Internet auctions with the fixed end time. Subsection 2.2 presents possible equilibria. Section 3 analyzes sequential Internet auction with flexible end times. Subsection 3.1 refines the model defined in subsection 2.1 to allow for the flexible end time. Subsection 3.2 studies possible equilibria. Section 4 discusses the plausibility of the presented equilibria. Finally, section 5 concludes.

2 Internet auctions with fixed end times

2.1 The model

There are two sequential auctions and $N > 2$ buyers. Each buyer i ($i = 1, ..., N$) has an independent private valuation of one (and only one) item of the good (v_i), which is distributed according to distribution $F(v_i)$ with density $f(v_i)$ and support on $[0, 1]$. $v^{m:n}$ denotes the m^{th} highest valuation out of n bidders. $F^{m:n}(.)$ is distribution of $v^{m:n}$. $f^{m:n}(.)$ is corresponding density.

Each auction a ($a = 1, 2$) takes two rounds. In each round t ($t = 1, 2$) of the auction a, the following happens[1]:

1. The *current price* (p_a^t) is announced.

2. Bids are collected from the *potential buyers*.

3. *Active bidders* and the *current winner* are indicated.

[1] There are only two rounds to make the argument clear. All the interesting trade-offs are illustrated.

A buyer i is a *potential buyer* in the auction a, if he has not won the good in the preceding auction. In each round t of the auction a, he can submit a positive bid $b^t_{a,i} > 0$ or not to bid at all ($b^t_{a,i} = 0$). If he overbids the current price and his previous bids submitted in this auction, he becomes an *active bidder.* If he submits the highest bid in the auction a as the first one, he becomes a *current winner.* In case of a tie, the current winner is chosen randomly. The *current price* equals the highest bid submitted by the buyer who is not the current winner. If there is only one active buyer, the current price is zero. After the last round of the auction a, the current winner wins the good and quits the game. The final price (p_a) is chosen on the same basis as the current price.

Auctions are run in a sequence. The auction 2 starts, after the auction 1 ends. After the auction 2 finishes, the game ends. The utility of the buyer i is given by:

$$u_i(.) = \begin{cases} v_i - p_a & \text{if } i \text{ is the final winner in the auction } a \\ 0 & \text{if } i \text{ does not win any good} \end{cases}$$

Let $h^t_a \in H^t_a$ be a history of the learned outcomes (i.e. current winners, current prices, identity of buyers whose bids equal to current prices and identities of all other active buyers) up to stage t of the auction a. Then, an action of the buyer i at the stage t of the auction a is given by $b^t_{a,i}(v_i, h^t_a) \in R_+$. The strategy of the buyer i is described by the function:

$$\sigma_i = \begin{bmatrix} b^1_{1,i}(v_i, h^1_1) & b^2_{1,i}(v_i, h^2_i) \\ b^1_{2,i}(v_i, h^1_2) & b^2_{2,i}(v_i, h^2_2) \end{bmatrix} : [0,1] \times H \rightarrow R^4_+$$

where H is the space of all possible histories. Ω_i is the space of all possible σ_i. The prior belief is that each valuation is distributed according to $F(.)$. $\mu_i \in M_i$ is the updating rule the buyer i uses to update the belief on the valuations of the opponents, given the history. The final outcome, including final prices and final winners for all the auctions, is given by $o \in O$. The outcome function is $\varpi : \Omega \times [0,1]^N \rightarrow O$, where $\Omega = \Omega_1 \times \cdots \times \Omega_N$. The game is defined by $\Gamma_1 = \{\Omega_1, ...\Omega_N, M_1, ..., M_N, o(.)\}$.

2.2 Results

Suppose that every buyer i bids $b(v_i)$ st. $b'(v_i) > 0$ in the auction 1. The current winner wins the object for the price of $b(v^{2:N})$. All the remaining buyers bid their valuations in the second auction. Then, the resulting outcome is exactly the same as in two second-price sealed-bid private-value sequential auctions. Furthermore, when bidding $b(v_i)$, the buyer i uses the prior beliefs. Hence, the optimal bidding function is given by: $\hat{b}(v_i) = E[v^{3:N}|v^{2:N} = v_i]$, which is the optimal bidding function in the first out of the two sequential second-price sealed-bid independent private-value auctions (see Milgrom and Weber, 1982, and Weber, 1983).

When every buyer i bids $\hat{b}(v_i)$ in the round 1 of the auction 1, some of the buyers other from the current winner might be interested in bidding above $b(v^{2:N})$ in the round 2 of the auction 1. To kill their incentives, the current winner needs to increase his bid up to his valuation. When he does so, the other buyers have no chance for a positive transaction in the auction 1. It is then their best reply not to bid in the round 2 of the auction 1. The proposition 1 uses this intuition to construct a perfect Bayesian equilibrium.

Proposition 1 *Let $\hat{\sigma}_i$ be defined as follows:*

1. *the buyer i bids $\hat{b}(v_i) = E[v^{3:N}|v^{2:N} = v_i]$ in the round 1 of the auction 1,*
2. *if he wins, he bids v_i in the round 2 of the auction 1,*
3. *if he is a potential buyer in the auction 2, he bids v_i in the round 1 of the auction 2,*
4. *otherwise, he does not bid,*

Let $\hat{\mu}_i$ be such that, after seeing $b^1_{j,1}$, the buyer i believes that $v_j = \hat{b}^{-1}(b^1_{j,1})$ for every $j \in \{1, ..., N\}/\{i\}$. Then, $(\hat{\sigma}_1, ..., \hat{\sigma}_3; \hat{\mu}_1, ..., \hat{\mu}_3)$ is a perfect Bayesian equilibrium (in undominated strategies) of Γ_1. The equilibrium outcome is efficient. The expected price equals to $E[v^{3:N}]$ in both auctions.

Proof. See the Appendix. ■

In the equilibrium presented by the proposition 1, the buyer i is supposed to bid $\hat{b}(v_i)$ in the round 1 of the auction 1. In the beginning of the round 2 of the auction 1, he does not know whether $b^1_{j,1} = \hat{b}(v_j)$ for every $j \neq i$. Hence, in the round 2 of the auction 1, he should have exactly the same updating rule, no matter what the opponents do in the round 1 of the auction 1. Since the Bayesian updating implies $v_j = \hat{b}^{-1}(b^1_{j,1})$ for every $j \neq i$, the buyer i should always believe that $v_j = \hat{b}^{-1}(b^1_{j,1})$ for every $j \neq i$ in the beginning of the round 2 of the auction 1. Hence, already in the beginning of the round 2 of the auction 1, he learns all the valuations. It would be strange if he changed his beliefs later on. Therefore, he is supposed to believe that $v_j = \hat{b}^{-1}(b^1_{j,1})$ for every $j \neq i$ after the round 1 of the auction 1 till the end of the game.

Given the beliefs, the behavior presented in the proposition 1 is sequentially rational. In the auction 2, every potential buyer weakly prefers to bid the valuation, no matter what believes he has. In the round 2 of the auction 1, the losing buyers do not bid, because they know that the bid higher than their valuations will be submitted there. The current winner bids, because he knows that his bid does not affect the price. Finally, in the round 1 of the auction 1, buyers use prior beliefs. Hence, the argument from the sequential sealed-bid auctions easily applies.

In the equilibrium presented in the proposition 1, the crucial bids are submitted in the round 1 of the auction 1. The proposition 2 presents a perfect Bayesian equilibrium in which buyers submit their crucial bids in the round 2 of the auction 1.

Proposition 2 *Let $\bar{\sigma}_i$ be defined as follows:*

1. *if $p^1_2 = 0$, the buyer i bids $\hat{b}(v_i) = E[v^{3:N}|v^{2:N} = v_i]$ in the round 2 of the auction 1,*
2. *if $p^1_2 > 0$, he bids v_i in the round 2 of the auction 1,*
3. *if he is a potential buyer in the auction 2, he bids v_i in the round 1 of the auction 2,*
4. *otherwise, he does not bid,*

Let $\bar{\mu}_i$ be as follows:

1. *if $p^1_2 = 0$, the buyer does not update his beliefs,*
2. *if $p^1_2 > 0$, the buyer i believes that $v_i < v^{2:N}$,*
3. *otherwise, he uses a Bayes rule, whenever possible,*

Then, $(\bar{\sigma}_1, ..., \bar{\sigma}_N; \bar{\mu}_1, ..., \bar{\mu}_N)$ is a perfect Bayesian equilibrium (in undominated strategies) of Γ_1. The equilibrium outcome is efficient. The expected price equals to $E[v^{3:N}]$ in both auctions.

Proof. See Appendix. ■

In the equilibrium presented in the proposition 2, no-one is supposed to bid in the round 1 of the auction 1. Then, the current price in the round 2 of the auction 1 is zero ($p_2^1 = 0$) and no new information is revealed. Therefore, on the equilibrium path, it is optimal for every buyer i to bid $\hat{b}(v_i)$ in the round 2 of the auction 1, just in the equilibrium presented in the proposition 1. The buyer with the highest valuation wins the good for the price of $\hat{b}(v^{2:N})$. Out of the equilibrium, when the current price in the round 2 of the auction 1 is positive ($p_2^1 > 0$), the buyer i believes that $v_i < v^{2:N}$. Knowing that every potential buyer bids the valuation in the auction 2, he realizes that he has no chance for a positive transaction in the auction 2. Hence, he weakly prefers to bid the valuation in the round 2 of the auction 1. In the auction 2, each potential weakly prefers to bid the valuation, both on and off the equilibrium path. The buyer with the second highest valuation wins the good for the price of $v^{3:N}$ there. The equilibrium leads to the same outcome as the equilibrium presented in the proposition 1.

The equilibrium with sniping is supported by specific off the equilibrium beliefs. These beliefs allow to deal with all the possibilities out of the equilibrium in an elegant manner. They imply the intuitive idea of the fear of causing the bidding war by early bidding, which an often mentioned motive of the on-line bidders (see Roth and Ockenfels, 2002 and forthcoming, for a discussion).

To conclude, the present subsection shows that there are multiple perfect Bayesian equilibria in undominated strategies. It is easy to construct other equilibria by changing the behavior and the beliefs off the equilibrium path. The point is that in a perfect Bayesian equilibrium in undominated strategies, buyers might bid early or snipe. No matter when exactly they decide to bid in the auction 1, they can still reach the market outcome.

3 Internet auctions with flexible end times

3.1 The model

There are again two sequential auctions and $N > 2$ buyers. Each buyer i ($i = 1, ..., N$) has an independent private valuation v_i that satisfies the condition imposed in the subsection 2.1. Auctions are run in a sequence. The auction 2 starts, after the auction

1 ends. After the auction 2 finishes, the game ends. The utility of the buyer i is given as in the subsection 2.1.

Each auction a ($a = 1, 2$) has $T_a + 1$ rounds. In each round t ($t = 1, ..., T_a + 1$) of the auction a, first, the *current price* is announced; second, bids are collected from the *potential* buyers and third, *active bidders* and the *current winner* are indicated, where the current price, bids, potential buyers, active buyers and current winners are defined as in the subsection 2.1. The round $T_a < \infty$ is determined endogenously. It is the last round with accepted bids. $T_a = 2$, if there are no bids in the round 2 of the auction a. If there is a bid in the round 2, the auction is extended for one more round. If there is also a bid accepted in the round 3, the auction is extended for one more round. More generally, if there is a bid accepted in every round $t \in \{2, ..., t^*\}$ and there is no bid accepted in the round $t^* + 1$, then $t^* = T_a$.

Let $h_a^t \in H_a^t$ be a history of the learned outcomes (i.e. current winners, current prices, identity of buyers whose bids equal to current prices, identities of all other active buyers and number of rounds) up to stage t of the auction a. Then, an action of the buyer i at the stage t of the auction a is given by $b_{a,i}^t(v_i, h_a^t) \in R_+$. The strategy of the buyer i in the auction a is described by the function: $\sigma_{a,i} = \left[\begin{array}{ccc} b_{a,i}^1(v_i, h_a^1) & ... & b_{a,i}^{T_{a,i}}(v_i, h_a^{T_{a,i}}) \end{array} \right] :$ $[0, 1] \times H_a^t \to R_+^{T_{a,i}}$. $T_{a,i}$ is the last round of the auction a in which, given some history and the valuation, the buyer i submits a positive bid. Since $T_a < \infty$, $T_{a,i} < \infty$. Let $\sigma_i = \left[\begin{array}{cc} \sigma_{1,i} & \sigma_{2,i} \end{array} \right] \in \Omega_i$. The beliefs are defined as in the subsection 2.1. $o \in O$, ϖ and Ω have the same meaning as in the subsection 2.1. The game is defined by $\Gamma_2 = \{\Omega_1, ...\Omega_N, M_1, ..., M_N, o(.)\}$.

3.2 Results

Incentives of the buyers are similar as in the model presented in section 2. In the auction 2, every potential buyer bids the valuation. In the auction 1, each buyer i first bids $b(v_i)$ which is strictly increasing in v_i. When all the bids but the highest one are revealed, buyers learn the ordering of their valuations. The buyer i with $v_i < v^{2:N}$ learns that he has no chance to win the good at a profitable price in the auction 2. He weakly prefers to increase his bid up to v_i. He is indifferent between bidding v_i in the auction 1 and not bidding at all, if he knows that the bid higher than v_i is submitted there. In an equilibrium, the current winner bids the valuation in the auction 1 and no-one else bids any more. The outcome is efficient. The revenue equivalence implies that in the auction

1, the expected price is given by $Ev^{3:N}$, which is the case if $b(v_i) = E[v^{3:N}|v^{2:N} = v_i]$. All in all, equilibrium presented in the proposition 1 is an equilibrium under the flexible end time.

Proposition 3 *The strategies and beliefs defined in the proposition 2 form a perfect Bayesian equilibrium of the game* Γ_2*. The equilibrium outcome is efficient. The expected price equals to* $E[v^{3:N}]$ *in both auctions.*

The proposition 3 gives an example of an equilibrium with early bidding. It is interesting to check whether there exists an efficient symmetric equilibrium with late bidding. Since bidding in stage $T_a + 1$ is not allowed, late bidding has to correspond to bidding in stage T_a. In the auction 2, buyers are indifferent when to bid the valuation. There is an equilibrium in which they bid only in stage T_2. In the auction 1, each buyer i first bids $b(v_i)$, st. $b'(v_i) > 0$. This is necessary as otherwise buyers do not know the ordering of their valuations and hence, are unable to reach an efficient outcome. To have sniping, each buyer i needs to bid only $b(v_i)$ in the auction 1. Furthermore, everyone needs to bid in the same round. Then, in the next round, the auction 1 ends and there is sniping. The question is whether the buyer i has incentives to bid only once, if everyone else bids only once. To see the answer suppose that every buyer $j \neq i$ bids only $b(v_j)$ in the auction 1 and that the buyer i first bids $b(v_i)$ in the auction 1 and later acts optimally. Then, the buyer i with $v_i < v^{2:N}$ will bid only once, if $b(v_i) = v_i$. Otherwise, knowing that $E[p_2] > v_i$ and that p_1 is smaller than v_i with a positive probability, he prefers to bid v_i in the auction 1. Hence, $b(v_i) = v_i$ is the first condition for an efficient equilibrium with sniping. The second condition is implied by the revenue equivalence theorem and states that in an efficient equilibrium, $E[p_1] = E[v^{3:N}]$. Clearly, the two conditions do not hold simultaneously. Thus, there is no equilibrium with sniping in the auction 1.

Proposition 4 *There is no efficient symmetric pure perfect Bayesian equilibrium (in undominated strategies) with sniping. That is, in an efficient symmetric pure perfect Bayesian equilibrium (in undominated strategies) of* Γ_2*, there is at least one buyer who bids twice in the auction 1.*

To conclude, there is no equilibrium in which buyers snipe. There is an efficient equilibrium with early bidding. It is easy to construct another equilibrium, for example by adding irrelevant bids.

4 The discussion on the existing equilibria

4.1 Definitions

The games defined in the subsections 2.1 and 3.1 are complex. There are multiple efficient perfect Bayesian equilibria in undominated strategies. The multiplicity occurs, because in the presence of many possible beliefs, the set of undominated strategies is wide. However, restricting attention only to the given belief dramatically shrink the set of possible strategies of the buyers. To show the point, I introduce the concept of an iteratively undominated strategy conditioned on the belief and I restrict my attention to a perfect Bayesian equilibrium in these strategies.

Definition 5 *A strategy* $\sigma_i \in \Omega_i$ *is an iteratively undominated strategy conditioned on* $\{\mu_1, ..., \mu_N\}$*, if it is an iteratively undominated strategy in the game* $\{\Omega_1, ...\Omega_N, \mu_1, ..., \mu_N, o(.)\}$*.*

Definition 6 $(\sigma_1, ..., \sigma_N; \mu_1, ..., \mu_N)$ *is an equilibrium if it is a symmetric perfect Bayesian equilibrium and for every* $i \in \{1, ..., N\}$*,* σ_i *is an iteratively undominated strategy conditioned on* $\{\mu_1, ..., \mu_N\}$*.*

4.2 Results

The previous sections presented multiple efficient perfect Bayesian equilibria that are strategically different, but outcome-equivalent. Hence, it seems that Internet auctions implement the market outcome very well. However, as I will argue below, there is no efficient equilibrium with early bidding. In an efficient equilibrium, buyers always snipe.

In an efficient equilibrium, the buyer with the highest valuation wins the good in one of the two auctions. At the beginning of the game, no-one knows who has the highest valuation. Buyers need to bid so that they learn the information from the revealed bids. When they bid, there is a buyer i who learns that $v_i < v^{2:N}$. Suppose he learns it in the round 1 of the auction 1. In an undominated strategy, each potential buyer bids the valuation in the auction 2. Hence, the final price will be at least given by $v^{2:N}$. This means that the buyer i with $v_i < v^{2:N}$ weakly prefers to bid v_i in the auction 1.

Lemma 7 *Suppose that* $p = \{p_1, ..., p_N\}$ *and* h_1^1 *are such that the buyer* i *learns that* $v_i < v^{2:N}$*. Then, in every iteratively undominated strategy conditioned on* p*, the buyer* i *bids* v_i *in the round 2 of the auction* 1*.*

Proof. See Appendix. ■

The lemma 7 implies that the perfect Bayesian equilibrium presented in the proposition 1 and in the proposition 3 does not satisfy the conditions imposed by the definition 6. It does not immediately imply that there is no efficient equilibrium with bidding in the round 1 in the auction 1. To verify it, I need to check whether there exists some optimal bidding function for the round of the auction 1. Suppose that all the buyers bid according to $\bar{b}(.)$, where $\bar{b}'(.) > 0$, in the round 1 of the auction 1 and that every buyer i with $v_i < v^{2:N}$ bids v_i in the round 2 of the auction 1. Then, the expected final price in the auction 1 is at least given by:

$$p^* = E[b(v^{2:N})] \cdot \Pr[b(v^{2:N}) > v^{3:N}] + E[v^{3:N}] \cdot \Pr[b(v^{2:N}) < v^{3:N}]$$

Clearly, $p^* > E[v^{3:N}]$. Hence, by the Revenue Equivalence Theorem, it is not an efficient equilibrium. The following theorem shows that in an efficient equilibrium, buyers always snipe.

Theorem 8 $(\bar{\sigma}_1, ..., \bar{\sigma}_N; \bar{\mu}_1, ..., \bar{\mu}_N)$ *as defined in the proposition 2 is an equilibrium of* Γ_1*. In an efficient equilibrium of* Γ_1*, in the auction 1, buyers always wait with bidding for the round 2. There is no efficient equilibrium of* Γ_2*.*

Proof. See Appendix ■

To conclude, to discuss the role of the ending rule in Internet auctions, I presented a model that had multiple efficient Prefect Bayesian equilibria under both the fixed end rule and the flexible end rule. Then, I restricted the set of possible strategies. These strategies are similar to undominated strategies and impose some rationality on the buyers. In the restricted set of strategies, I identified possible efficient perfect Bayesian equilibria. It turned out that there is no equilibrium with early bidding. In an equilibrium, buyers always snipe. This means that the fixed end time is necessary to have the market equilibrium. Under the flexible end time, the equilibrium does not exist. Hence, having the fixed end time instead of the flexible end time is better for efficiency.

5 Conclusions

The markets where the demand meets the supply do not exist. Yet, many transactions take place every second. More and more of these transactions take place on Internet auctions. Internet enables trade on a longer distance. It does not require the seller and the potential buyers to be available in the same moment. Therefore, Internet auctions are dynamic and have no activity rules. When having dynamic auctions without activity rules, the auction platform needs to decide upon the ending rule. However, they are offered little guidelines from the auction theory. The present paper studies the optimal ending rule from the perspective of the auction platform which is interested in the market clearing. It argues that it is difficult to have the market outcome under the flexible end rule. Under the fixed rule, the market outcome is easily obtained. Hence, the fixed end time is more desirable than the flexible end time.

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Appendix

Proposition 1: *Let $\hat{\sigma}_i$ be defined as follows:*

1. *the buyer i bids $\hat{b}(v_i) = E[v^{3:N}|v^{2:N} = v_i]$ in the round 1 of the auction 1,*
2. *if he wins, he bids v_i in the round 2 of the auction 1,*
3. *if he is a potential buyer in the auction 2, he bids v_i in the round 1 of the auction 2,*
4. *otherwise, he does not bid,*

Let $\hat{\mu}_i$ be such that, after seeing $b^1_{j,1}$, the buyer i believes that $v_j = \hat{b}^{-1}(b^1_{j,1})$ for every $j \in \{1, ..., N\}/\{i\}$. Then, $(\hat{\sigma}_1, ..., \hat{\sigma}_3; \hat{\mu}_1, ..., \hat{\mu}_3)$ is a perfect Bayesian equilibrium (in undominated strategies) of Γ_1. The equilibrium outcome is efficient. The expected price equals to $E[v^{3:N}]$ in both auctions.

Proof. If (1) not having updated the beliefs, each buyer i bids $\hat{b}(v_i)$ in the auction 1, (2) the final price is given by $\hat{b}(v^{2:N})$ in the auction 1, (3) the final price is given by $v^{3:N}$ in the auction 2 and (4) the outcome is efficient, then $\hat{b}(v_i)$ equals to $E[v^{3:N}|v^{2:N} = v_i]$, which is the optimal bidding function in the first out of the two sequential second-price sealed-bid private-value auctions (see Milgrom and Weber, 1982, and Weber,1983, for the results on sequential second-price sealed-bid private-value auctions). The strategy profile is constructed so that (1)-(4) hold. Hence, $\hat{b}(v_i) = E[v^{3:N}|v^{2:N} = v_i]$. Furthermore, as Milgrom and Weber (1982) and Weber (1983) show, $E[p_1] = E[p_2] = E[v^{3:N}]$. In the auction 2, every potential buyer weakly prefers to bid the valuation. The beliefs are constructed so that on the equilibrium path the Bayes rule is used. If the buyer i is not a current winner in the round 1 of the auction 1, he knows that the highest bid submitted by his opponents in the auction 1 equals to $v^{1:N-1}$ and the highest bid submitted by his opponents in the auction 2 equals to $v^{2:N-1}$ or $v^{1:N-1}$. Hence, he weakly prefers not to bid in the round 2 of the auction 1, to bid v_i in the round 1 of the auction 2 and not to bid in the round 2 of the auction 2. If the buyer i is the current winner in the round 1 of the auction 1, he knows that no-one else is supposed to bid in the round 2 of the auction 1. Hence, he weakly prefers to bid v_i in the round 2 of the auction 1. Besides following 3. of $\bar{\sigma}_i$, he has no reason to bid any more. All in all, $(\bar{\sigma}_1, ..., \bar{\sigma}_N; \bar{\mu}_1, ..., \bar{\mu}_N)$ is a perfect Bayesian equilibrium (in undominated strategies) of Γ_1. The equilibrium outcome is efficient. The expected price equals to $E[v^{3:N}]$ in both auctions. ■

Proposition 2: *Let $\bar{\sigma}_i$ be defined as follows:*

1. *if $p_2^1 = 0$, the buyer i bids $\hat{b}(v_i) = E[v^{3:N} | v^{2:N} = v_i]$ in the round 2 of the auction 1,*
2. *if $p_2^1 > 0$, he bids v_i in the round 2 of the auction 1,*
3. *if he is a potential buyer in the auction 2, he bids v_i in the round 1 of the auction 2,*
4. *otherwise, he does not bid,*

Let $\bar{\mu}_i$ be as follows:

1. *if $p_2^1 = 0$, the buyer does not update his beliefs,*
2. *if $p_2^1 > 0$, the buyer i believes that $v_i < v^{2:N}$,*
3. *otherwise, he uses a Bayes rule, whenever possible,*

Then, $(\bar{\sigma}_1, ..., \bar{\sigma}_N; \bar{\mu}_1, ..., \bar{\mu}_N)$ is a perfect Bayesian equilibrium (in undominated strategies) of Γ_1. The equilibrium outcome is efficient. The expected price equals to $E[v^{3:N}]$ in both auctions.

Proof. On the equilibrium path the following is satisfied: (1) not having updated the beliefs, each buyer i bids $\hat{b}(v_i)$ in the auction 1, (2) the final price is given by $\hat{b}(v^{2:N})$ in the auction 1, (3) the final price is given by $v^{3:N}$ in the auction 2 and (4) the outcome is efficient. Hence, $\hat{b}(v_i)$ equals to $E[v^{3:N}|v^{2:N} = v_i]$, which is the optimal bidding function in the first out of the two sequential second-price sealed-bid private-value auctions (see Milgrom and Weber, 1982, and Weber,1983, for the results on sequential second-price sealed-bid private-value auctions). What's more, $E[p_1] = E[p_2] = E[v^{3:N}]$. On the equilibrium path, buyers use the Bayes rule to update the beliefs. In the auction 2 everyone weakly prefers to bid the valuation. It remains to prove that no-one has incentives to bid in the round 1 of the auction 1 and that, if $p_2^1 > 0$, the buyer i bids v_i in the round 2 of the auction 1. If the buyer i is the only bidder in the round 1 of the auction 1, the other buyers do not change their behavior and hence, the buyer i does not gain anything. If there is at least one other buyer in the round 1 of the auction 1, the situation of the buyer i worsens, as his opponents become more aggressive. Hence, he has no incentives to bid in the round 1 of the auction 1. If $p_2^1 > 0$, the buyer i believes that $v_i < v^{2:N}$. Since all the potential buyers bid their valuations in the auction 2, he will not win the good at a profitable price in the auction 2. Hence, he weakly prefers to bid v_i in the round 2 of the auction 1. ■

Proposition 3: *The strategies and beliefs defined in the proposition 2 form a perfect Bayesian equilibrium of the game* Γ_2. *The equilibrium outcome is efficient. The expected price equals to* $E[v^{3:N}]$ *in both auctions.*
Proof. The proof follows straightforwardly from the proposition 2. ■

Proposition 4: *There is no efficient symmetric pure perfect Bayesian equilibrium (in undominated strategies) with sniping. That is, in an efficient symmetric pure perfect Bayesian equilibrium (in undominated strategies) of* Γ_2, *there is at least one buyer who bids twice in the auction 1.*
Proof. The proposition 3 shows that there is an equilibrium with one buyer bidding twice in the auction 1. It remains to prove that there is no equilibrium in which every buyer bids only once in the auction 1. Suppose the opposite. Let each buyer i bid $b(v_i)$ in the auction 1 and do not bid any more in the auction 1. Since the equilibrium is efficient, $b'(v_i) > 0$, so that the buyer with the highest valuation wins the good in the auction 1. By the revenue equivalence theorem, $E[p^1] = E[b(v^{2:N})] = E[v^{3:N}]$. Hence, $b(v_i) < v_i$. If so, the buyer i with $v_i < v^{2:N}$ prefers to bid v_i in the auction 1. A contradiction. ■

Lemma 7: *Suppose that* $p = \{p_1, ..., p_N\}$ *and* h_1^1 *are such that the buyer* i *learns that* $v_i < v^{2:N}$. *Then, in every iteratively undominated strategy conditioned on* p, *the buyer* i *bids* v_i *in the round 2 of the auction* 1.

Proof. In an undominated strategy, every potential buyer bids the valuation in the auction 2. Hence, the final price is at least given by $v^{2:N}$. Therefore, the buyer i who learns that $v_i < v^{2:N}$ knows that he will not have any positive transaction in the auction 2. If so, he weakly prefers to bid the valuation in the round 2 of the auction 1. ■

Theorem 8: $(\bar{\sigma}_1, ..., \bar{\sigma}_N; \bar{\mu}_1, ..., \bar{\mu}_N)$ *as defined in the proposition 2 is an equilibrium of* Γ_1. *In an efficient equilibrium of* Γ_1, *in the auction 1, buyers always wait with bidding for the round 2. There is no efficient equilibrium of* Γ_2.

Proof. To prove that the symmetric perfect Bayesian equilibrium presented in the proposition 2 is an equilibrium defined in the definition 6, it is sufficient to prove that the strategies are iteratively undominated condition the beliefs. In the auction 2, it is an undominated strategy to bid the valuation. Hence, in the auction 2, the potential buyer i competes against the bid that is at least given by $v^{2:N-1}$ and at most given by $v^{1:N-1}$. If so, it is his iteratively undominated strategy condition on not updating the belief to bid $E[v^{2:N-1}|v^{1:N-1} = v_i] = E[v^{3:N}|v^{2:N} = v_i]$. If he believes that $v^{2:N} > v_i$, then it is his iteratively undominated strategy to bid v_i in the round 2 of the auction. In the round 1 of the auction 1, it is his iteratively undominated strategy not to bid. This is because, given the prior belief, this strategy performs the best, when the other buyers increase their bids, when seeing the bids in the round 1 of the auction 1. Hence, the perfect Bayesian equilibrium presented in the proposition 2 is an equilibrium defined in the definition 7.

Now, I prove that in an efficient equilibrium of Γ_1, in the auction 1, buyers always wait with bidding for the round 2 of the auction 1. Suppose otherwise. In a symmetric efficient equilibrium with early bidding, every buyer i bids $b(v_i)$ in the round 1 of the auction 1. To have early bidding, I need to assume that $b(v_i) > 0$ for some $v_i \in [\underline{v}, \bar{v}]$, where $\underline{v}$,$\bar{v} \in [0, 1]$ and $\underline{v}$<$\bar{v}$. Since, in undominated strategy, the buyer i never bids above the valuation, $b(v_i) \leq v_i$ for every v_i. This implies that whenever $p_1^2 > v_i$, the buyer i learns that $v_i < v^{2:N}$. If so, by lemma 7, he bids v_i in the round 2 of the auction 2. Then, in the auction the final price is at least given by $\max[b^*, v^{3:N}]$, where b^* is the second order statistic of $(b(v_1), ..., b(v_N))$. Since $b^* > 0$ with positive probability, $E[p_2] \geq E[\max[b^*, v^{3:N}]] > E[v^{3:N}]$. But the revenue equivalence theorem, this is not possible in an equilibrium. A contradiction.

Now, I prove that there is no efficient equilibrium of Γ_2. If have just proved that there is no equilibrium with bids in the round 1. It is thus sufficient to show that there is no equilibrium with bids in the round 2. Suppose the opposite. Let each buyer i bid $b(v_i)$ in the round 1 of the auction 1. Again, $b(v_i) > 0$ for some $v_i \in [\underline{v}, \bar{v}]$, where $\underline{v}$,$\bar{v} \in [0, 1]$

and $\underline{v}<\bar{v}$, and $b(v_i) \leq v_i$ for every v_i. Hence the buyer i with $v_i < p_1^2$ learns that he will not win the good for a profitable price in the auction 2. Hence, he weakly prefers to bid v_i in the round 3 of the auction 1. But then, the expected final price is too high. A contradiction. ■